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Turtle's Navigation and Manipulation of Geometrical Figures Constructed by Variable Processes in a 3d Simulated Space

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Abstract. Issues related to 3d turtle's navigation and geometrical figures' manipulation in the simulated 3d space of a newly developed computational environment, MaLT, are reported and discussed here. The joint use of meaningful formalism and the dynamic manipulation of graphically represented 3d figures seem to offer new resources and to pose new challenges as far as geometrical activities and construction of meanings are concerned, which are strongly related to the representational infrastructure of MaLT.

Abilities such as spatial orientation and spatial visualisation come into play and are interwoven with the software's functionalities and semantics. Although the body-syntonic metaphor remains critical while navigating the turtle in the 3d simulated space, it seems that it has to be co-ordinated with other – often conflicting one another – frames of reference. The strong link between spatial graphical and geometrical aspects, that was accentuated by the dragging functionalities of the software, helped students go beyond an immediate perceptual approach, relating geometrical figures with real 3d objects and the change of planes in 3d space with physical angle situations. In this framework the concept of angle as turn and measure with emphasis on directionality but also as a relationship between the planes defined by 2d figures has arisen as central.

Key words: 3d logo, dynamic manipulation, spatial visualisation, frames of reference, angle.

Theoretical Background

In this paper we report a small-scale preliminary research aiming at shedding light upon issues related to turtle's navigation and geometrical figures manipulation that have arisen while 11 year-old students were carrying out collaboratively constructionist activities in the computational environment of a newly developed software, MaLT. MaLT was conceived as a constructionist microworld environment (Papert, 1980; Harel and Papert, 1991; Kafai and Resnick, 1996) for programmable constructions in 3d geometrical space. It is designed so as to extend the integrated 'programming-dynamic manipulation' (Kynigos, 2004) idea to 3d space, while most ICT tools (logo-based environments, DGEs) have been designed to operate within 2d environments.

The new kinds of representations offered (e.g., 3d turtle in a simulated 3d space), the new kinds of access offered to conventional representations (e.g., dynamic manipulation of the values of the variables of parametric procedures) and the way these changes affect the symbolic activity and the construction of mathematical meanings need investigation. So far the pedagogical value of graphical representations has been the object of big debate in the teaching of Geometry, since it is not clear exactly what a geometrical shape represents, an actual tangible figure or a class of ideal, but not real, objects. This is at the heart of geometry's dual nature as a field for abstract mathematical thinking or for understanding space (Laborde *et al.*, 2006). As far as geometry curriculum is concerned, in primary education emphasis is placed on formal symbolism and naming and not enough on spatial exploration, analysis and synthesis. Students learn of 3d geometry through presentation of static 3d objects, either in a concrete form or in printed form in their textbooks, without the chance of manipulating them.

Constructions in the context of MaLT could possibly bring in the foreground issues concerning the mathematical nature of 3d geometrical objects and how interactivity, control and experimentation in virtual reality microworlds can be a versatile vehicle for enhancing mathematically driven navigation, orientation and spatial visualization. Studying in a dynamic way 3d geometrical objects students have to analyse a 3d figure, break it into smaller parts and determine angle measures and lengths of line segments. Projecting themselves into the place of the turtle and moving from the visual to the descriptive level of thought students have to search for ways to reconceptualise 3d objects in terms that can be explained to the 3d turtle through logo commands. Moreover through the use of sliders students are provided with a direct manipulation metaphor for sequentially changing variables' values and simultaneously observing the variation both of 3d object and of their place in 3d space. As a result students have the chance to observe the behaviour of the varying parts in relation to each other and to the invariant ones and acquire a sense of generality and abstraction.

While interacting with each other and with the computational medium students come in contact with multiple linked, integrated and interdependent mathematical represen-

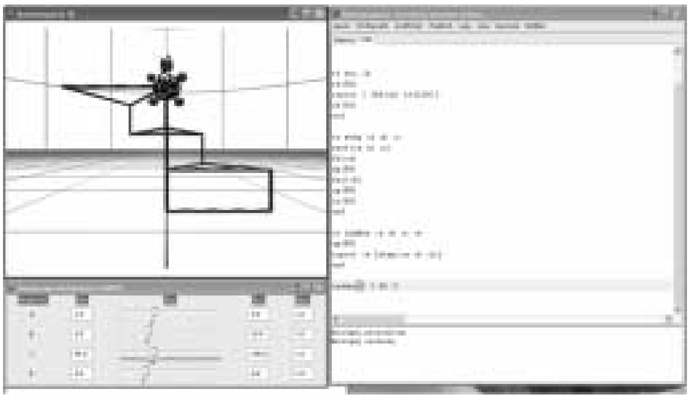

Fig. 1. The interface of MaLT.

tations. In particular students come in contact with three representational registers: a) meaningful formalism by means of symbolic programming b) graphical-figural representations in the simulated 3d space c) kinaesthetic activity and use of various interwoven but in many cases conflicting one another frames of reference (Wickens *et al.*, 2005). However an instrument does not exist in itself but only in relation to its utilisation by the user (Mariotti, 2001). As a result an instrument is not a fixed and permanent object but rather an evolving and versatile matter. For instance, in MaLT functionalities and phenomenological cues available are chosen by experts because they see on them an embodiment of something that they already know and understand (Kynigos and Latsi, 2006). The specific representations are media for what the experts already know. On the other hand for students these are only presentational models that may be used and conceptualized in quite a different way than the initially intended one. The construction of mathematical meanings as a result of students' interaction with a specific learning environment raises (among the others) semiotic issues related to screen phenomenology and semantics. Thus a careful analysis of the utilisation schemes developed and representational models used is needed so as to investigate the interplay between mathematical meanings and computer phenomenology. The interplay of the aforementioned representational registers in the process of knowledge construction viewed from an interactionist perspective will be the focus of our research interest.

Research Setting and Method

The Technology Used

Machine-Lab is a programmable environment for the creation and exploration of interactive virtual reality simulations developed within the ReMath project. Machine-Lab is implemented over the 3impact game engine (`www.3impact.com`) which wraps the stunning 3D graphics of the Direct3D engine and the distributed/multi-user capabilities of the DirectPlay engine, together with a 3D sound engine (with Doppler effects) and the Open Dynamics Engine (`www.ode.org`), which is a Rigid-Body Simulation engine with realistic physics support for 'collidable' objects and articulated bodies (rag-dolls / humanoids). The version of MaLT that was used consisted of tree components namely:

- *Turtle Scene* (TS). The main part of this component is a three-dimensional grid-like interface. A perspective projection is used with three active vanishing points, so that a realistic effect of 3d space representation and navigation is created. The 3d space representation is based on the metaphor of hemisphere. The two active vanishing points have a deviation of 90o and they are located in an iconic line of horizon which is conceived as the circumference of vertical circle. The third active vanishing point is conceived as the pole of the hemisphere. The turtle, which is a 3d object of .x type, is java-based and compiled so that the response from 3d graphics is fast. It can be navigated in the 3d simulated space providing rich representations of geometric objects and behaviours.

- *Logo Editor* (LE). This component is the Logo-like programming interface that engages students in logical procedural thinking and it is linked to the Turtle Scene. The user is able to write, run and edit Logo programs.
- *Uni-dimensional Variation Tool* (1dVT). The main part of this component consists of ‘number-line’-like sliders, each corresponding to one of the variables used in a Logo procedure.

Research Context

The research was carried out at the computer laboratory of a primary school in Athens with four sixth grade students divided in two groups. It should be noted that students didn’t have any experience with any programming language or with any dynamic geometry environment either 3d or 2d. Two tasks were carried out by students: in the first task, which was introductory, students were firstly asked to move the turtle in the right and left corner of the 3d space and then to bring it back at its initial position. The researcher didn’t specify on purpose what they meant with the left and right corner, so as to leave students explore freely the 3d space and develop their own ‘navigation’ strategies. In the second task students were asked to develop a parametric procedure that constructs a parallelogram and then to change its position in 3d space using the 1d Variation Tool.

Research Method

In our research we used a design-based research method (Cobb *et al.*, 2003) which entailed the ‘engineering’ of tools and task, as well as the systematic study of the forms of learning that took place within the specific context as defined by the means of supporting it. Initially the aim was to develop the software and the task in order to improve the educational process and to bring about new forms of learning, based on prior research and our theoretical framework. In retrospect, the research method that we used aims both at improving the initial design and at resulting in a situated understanding of the relationship among theory, designed artifacts and practice.

Espousing the role of naïve and participant observer the researchers took a generative stance allowing for the data to structure the results and to pilot their analysis. It should be stressed that we are not aiming at final and self – evident answers but at focusing better and at reformulating our initial questions. A team of two researchers participated in each data collection session using one camera and two tape-recorders. One researcher was occasionally moving the camera to all groups to capture the overall activity and other significant details as they occurred. Background data were collected (e.g., students worksheets, observational notes) and all audio-recordings were analyzed verbatim.

In analyzing the data we firstly looked for instances where meanings related to the visualisation and conceptualisation of the computer simulated 3d geometrical space were expressed by the students. The identified illustrative episodes can be defined as moments in time which have characteristic bearing on pupil’s interaction with the available tools and mathematical meanings construction. We have focused particularly on the process by which implicit mathematical knowledge is constructed during shared student activity while being sensitive to the interconnection between the construction of personal meanings and standard mathematical discourse. However in certain cases word episodes were meaningless if they were not related to the sequences of actions that students carried out while constructing their procedures or more importantly if they were not related to the gestures that they used. Thus in the analysis that follows word episodes are accompanied by photos with indicative students' gestures.

Results

Moving Around in the 3d Space

Moving the turtle around in the 3d simulated space of MaLT seemed to necessitate a dynamic visualisation of angle, which integrates two schemes:

- turn as body movement in 3d space which inevitably involves directionality and
- turn as number-measure.

It should be noted that in MaLT there are 3 kind of turns: right/left turn in relation to turtle's trunk-vertical axis, rightroll/leftroll which moves the turtle around its trunk/vertical axis and uppitch/downpitch, which pitches the turtle's nose up and down The multifaceted nature of the concept of angle as well as the difficulties that children encounter in various contexts -and while working with 2d Logo- are well documented in the literature (Clements *et al.*, 1996; Keiser, 2004; Michelmore and White, 2000). Although in first thought it seems that in this kind of computational environment the concept of angle is approached in it's a more natural and intuitive aspect, analysis of our results showed that new kinds of challenges arise while navigating a 3d turtle in the 3d simulated space.

St1: No, as it is from here, it turns that way and goes left.
St2: It goes that way, there is left.
St1: With the face of the turtle. . . it should come here.
St2: So left, look at it a bit.
St1: Should I correct it..Rt..we proceeded below.

From the above extract, as it was illuminated by students gestures, it was evident that students oscillated between different frames of reference (Wickens *et al.*, 2005), when they were trying to visualize 3d turtle's motion on the computer screen, either in order to decide what commands they should give or in order to find out what has caused the unexpected turtle's movement. The frames of reference used by students could be categorized in five groups:

- a world frame: defined in terms of directions 'up' and 'down';
- an ego frame: defined in terms of the orientation of the trunk or location of the observer;
- a head frame: defined in terms of the orientation of the head;
- a vehicle frame: typically associated with the orientation of a moving entity, here the turtle;

Fig. 2. Showing at the screen using a display frame of reference.

- a display frame, defining the orientation and movement of information on a display.

All the above frames of reference were intermingled when students were trying to find out the right kind of turn. Initially an ‘ego-frame’ was projected to a ‘vehicle-frame’. In other words children drove the turtle in a body-syntonic way, projecting the orientation of their trunk to the orientation of the vehicle of the motion, the turtle. Moreover the standard ‘display-frame’, the computer screen, was interconnected with the ‘world-frame’ and the ‘ego-frame’ and was defined in relation to four points, the standard up/down direction and right/left, in relation to the viewing point of the observer. In the 3d motion of the turtle another frame of reference was also present, which we can call the ‘head-viewing’ frame. In literature the head-frame is defined in terms of the orientation of the head, which may sometimes be different from that of the ego-trunk. While moving in the 3d turtle-scene, in order to understand where right, left, up and down is, attention should be given to *‘where the turtle was looking at’* or *‘where the turtle’s face was’*, to use children’s expressions. So, many times students had to explain to each other where right was, using simultaneously their hands for right and left, moving their bodies around its axis to imitate turtle’s motion and then showing on the screen. It follows that students had to co-ordinate all those frames in order to visualise turtle’s motion. It should be also noted that all these frames often conflict one another. For instance, right could be defined in relation to the display screen and the standard viewing point of the observer (display frame) or the turtle’s trunk axis (ego-vehicle frame), or the turle’s view-point (head-viewing frame).

Moreover it could be added that, although 3d simulated space is closer to real life and every-day experiences, the body-syntonic metaphor is less strong in 3d turtle geometry than in 2d. In other words when we move in real 3d space the up and down directions are usually stable (although not when we turn upside-down) because of gravity. Moreover, we walk in a 2d horizontal plane while the 3d turtle moves in different planes in 3d space. For instance, we can easily simulate 2d turtle motion with our body but we cannot simulate 3d turtle’s motion. Thus, it seems that the body-syntonic frame in 3d space is shrunk in favour of the ‘vehicle frame’.

Another finding, relative to students’ difficulty in navigating in 3d space using the 3d body-syntonic metaphor, is their preference in working initially in the vertical plane using only the four standard commands of 2d Logo (right, left, forward, backward). As shown in the following word episode their navigation in the vertical plane seemed more convenient, closer to their point of view and quicker. Initially they didn't find any utility in having the turtle moving around its axis (rightroll or leftroll command) or pitching its nose up and down (uppitch and downpitch commands). Only after more experimentation did they realise that by rolling the turtle around its axis the position of right and left changes accordingly. Their preference in working on the vertical plane could be possibly explained by the fact that children were more accustomed to 2D representations of geometrical figures. Three-dimensional representations on the computer screen are quite new and possibly students are not yet accustomed to the conventions used so as to represent a 3D object in a 2D form to look 3D in nature (Lowrie, 2002). On the other hand if we accept the view of Dalgarno *et al.* (2002) that we understand 3D models through multiple 2D representations, maybe students had focused subconsciously on a simplified 2D way of visualising 3d space.

> R: I see that you use more the right and forward commands, don't you?
> St1:Yes.
> R: Is it easier? Why don't you use the other commands? You didn't need them?
> St1: No, we didn't need them.
> St2: It's closer and more convenient. You can see easier the turtle.
>
> St2: Why should we have the turtle moving around itself? There is no point in it.
> R: There is no point in moving around herself?
> St: No, it manages everything this way and it's easier.

It could be also said that in order to visualise turtle's motion and find concrete analogues in real space children used two perspectives: an intrinsic- internal one and an external one. An intrinsic-internal one was used in cases that children were trying to co-ordinate their ego-frame with the vehicle frame and thus to view the surrounding environment from inside, as the space immediately surrounding them, while trying to conceptualise it in three dimensions constructed out of the axes of their body (Tversky, 2005). In these cases they used their hands and bodies in order to concretize turtle's motion. However when they were manipulating the figures created by the turtle they adopted an external view, they viewed those objects from outside and they sometimes needed concrete 3d objects, as transitional ones between real space and simulated 3d space. In fact, in order to visualise the change of planes of a parallelogram as a result of the change

Fig. 3. Using hands so as to project an ego-frame to the vehicle-frame of reference.

Fig. 4. Using a concrete 3d object to help them foresee the changes of planes of the parallelogram in the screen.

of the initial position of turtle in 3d space, children didn't use only their bodies or their hands but also concrete 3d objects, in our case a cassette that they moved in 3d space.

As far as the aspect of angle as number-measure of a turn is concerned, it seemed that students could easily foresee the degree of change of turtle's position and of the change of plane of the parallelogram drawn by the 3d turtle using as benchmarks the standard measures of 180, 90 and 45 degrees, drawing apparently upon their experience in 2d geometry in their mathematics course, a finding which is in accordance to other researches with 2d Logo environments (Clements and Burns, 2000).

R: So if I delete this downpitch command, and run the procedure 'parallelogram' what is going to happen?
St1: It is going to be flat.
St2: Yes
R: Flat, what has happened with the downpitch command?
St2: It has raised it that way.
R: That way.
St1: It has descended it down.
R: If we command downpitch 45, how do you thing it will appear?
St1: Somewhere in the middle.
R: Somewhere in the middle. If we input 90?
St2: It is going to appear that way.
R: If we input 50?
St1: It will be not in the middle but somewhere above the middle.

Semantics of the Software

The aforementioned results cannot be analysed in all their dimensions independently from the tool set with which students' construction strategies are expressed. In particular here we are going to try to explain students' visualisation of the simulated 3d space and construction strategies relating them to software's semantics and affordances.

Although construction in one plane is a meaningful choice as it minimizes the variables involved (children use only the standard 2d logo commands), it is still to be explained why children preferred to construct their figures in the vertical level and not on the horizontal one which is closer to the 'flat earth model' (Driver *et al.*, 1994) that we experience everyday. Working with the software we realised that the vertical axis that students preferred coincides with the viewing axis of the display. As a result and using children's words from a previously presented word episode, working in this level, *'you can see easier the turtle'*. This finding is in accordance to other researches contacted with 3d computational tools (Kynigos and Latsi, 2006) in relation to a 3D representation of vectors in computer screen, which have also shown that children have focused in a simplified 2D representation and that they had extra difficulties in appreciating the third dimension concerning depth (z coordinate).

While designing the tool we were sensitive to the difficulties present in trying to represent a 3d object in a 2d form to look 3d in nature and we inserted a grid in the 'Turtle Scene' as a visual hint to help students appreciate depth. However it seems that it wasn't helpful enough as far as the third dimension is concerned, a point which should be taken into account in a further development of the software. To make matter worse, in cases where the trace of the turtle coincided with the lines of the grid the appreciation of the depth was more complicated. Moreover the fact that the grid followed the convention of the linear perspective and of a vanishing point visible in the line of horizon confused students as far as the course of the turtle in the z coordinate was concerned. Did the turtle follow a perpendicular route to the vertical level (called by the students as straight line in the following excerpt) or did it follow a slanted one?

St2: With the depth, I don't know. Should I input 5?
St1: Yes, do it.
R: Where is it? Is its trace obvious?
St1: No, it is more intense. It went inside, it didn't go straight.
R: Oh
St1: If this line weren't there, it would be visible.

Finally the affordance of manipulating dynamically and in a kinaesthetic way the values of the variables of their procedure offered students the opportunity to animate their constructions, while connecting the visual features of their constructions to their mathematical underpinnings. Moving their constructions around in 3d space and changing planes helped students acquire a sense of generality and abstraction underlying the mathematical concepts involved -such as the notion of angle in 3d space- which are usually presented in a static way. Angle was conceived not only as the turn of the turtle and its measure but also as a relationship between the planes defined by the 2d parallelogram drawn by the turtle. In the following excerpt it is evident that students could easily move their parallelogram in any direction and find its analogue to everyday physical 3d objects. It is interesting that children did not only connect their construction with a physical 3d object but also with physical angle situations, as the limited rotation of a door, which usually involves extra difficulties for students. As Michelmore and White (2000) points out, while opening and closing a door the main difficulty lay in identifying its initial position as one arm of the angle, as the standard angle concept first develops in situations where both arms of the angle are visible.

St2: It's like a door.
R: Like a door? How can we have it upright?

St2: I will move it vertically.
R: So you made it a door. Do you want it to turn this way?
St1: No, vertically. . . or this way.

Discussion

New representations enabled by MaLT can place spatial visualization concepts in a central role for both controlling and measuring the behaviours of entities and figures in virtual 3d environments. We find the ‘distance’ between this new representations and the conventional, static ones as a challenging point to provoke unexpected pupil’s responses while giving in parallel rise to new kinds of problems requiring geometrical knowledge. Three-dimentiomality in the computational environment of MaLT poses special challenges concerning abilities such as spatial navigation and spatial visualisation which seem to be interwoven with the use of various frames of reference and mathematical formalism.

In particular, while navigating the 3d turtle in the 3d space of MaLt or when manipulating dynamically geometrical figures it seems that the concept of angle as turn and measure with emphasis on directionality is central. Angle was approached more as a turn that is neither the static pair of sides nor the enclosed planar or spatial part but the process of change of direction and planes in 3d space. Moving around in 3d space involves the coordination of various frames of reference (Wickens *et al.*, 2005) which in many cases conflict one-another. Although the notion of body-syntonicity is critical, it is evident that it in intermigled with the ‘display’ frame of reference, the ‘vehicle’ frame of reference and the ‘head- face’ one. In other words in order to navigate in the 3d simulated space students have to co-ordinate their static view point of the 2d computer screen while projecting their knowledge about their bodies and how they move into the ‘vehicle’ of the motion, the turtle, whose heading-face has a special importance (In 2d turtle’s motion attention is paid only to turtle’s heading, while here attention should be given to turtle’s ‘face’, as the turtle can be rolled around its vertical axis). Moreover the ‘world’ frame of reference where the up and down are fixed as well as the fact that we move in a horizontal 2d plane are challenged and ‘the body-syntonicity’ metaphor has to be applied in a 3d turtle that can change planes and orientation without any limitation.

As a result students seemed to use two perspectives while they were trying to visualise turtle’s motion: an external view perspective and an internal one. The first perspective was used in cases that they were trying to navigate the turtle in 3d space and to see the 3d space from ‘inside’ conceptualising it in three dimensions constructed out of the axes of their bodies. In these cases they used their hands and bodies so as to concretize turtle’s motion. It should be noted that in contrast to researches conducted with 2d logo we didn’t observe a curtailment of physical strategies (Clements and Burns, 2000). Students didn’t proceed from large body movements to smaller ones, using initially their body and then their hands to help them visualise turtle’s motion. They used their hands to concretize the standard 2d Logo commands, and especially right and left, and their trunk in order to concretize the rightroll and leftroll commands. It is interesting that they didn’t use any body-movement to imitate the uppitch/downpitch commands, probably because pitching our nose up and down is not followed by movement of our trunk as it happens with 3d turtle. The 'external' view perspective was used in order to visualise the change of planes of a parallelogram as a result of the change of the initial position of the turtle in 3d space. In this case children viewed this object from outside and they used not their bodies but concrete 3d objects, as transitional ones between real space and simulated 3d space. In this framework an interesting point for a further research could be the integration in task design of an absolute frame of reference where the place of the turtle would be also defined by 3d coordinates, a functionality which is also available in MaLT.

Moving from the visual to the descriptive level of thought students had to search for ways to reconceptualise geometrical figures in terms of 3d logo commands. Translation of visual features into conceptual relations was facilitated by the dragging functionalities of the software that seemed to have acted as a mediator between figures on the one hand and mathematical formalism and concepts on the other (Kynigos, 2004). Dragging didn't provide just a kinesthetic sense of dynamic manipulation and animation of mathematical objects changing only the visual characteristics of the figures. It provided an action/notation context that rendered parametric procedures descriptors of evolving geometrical objects by means of dragging a slider and observing how the object changes in relation to the values of the variables. In this way the distinction between the conceptual part of the notion of angle as turn and measure in 3d space and the figural one was blurred unlike what is usually the case in geometrical tasks (Mariotti, 2001). Initiating from the use of a parallelogram as a class of ideal but not real object students passed to its use as an actual tangible figure, bridging geometry's dual nature (Laborde *et al.*, 2006). In this research a parallelogram was used as an external object that was connected to real-life experiences and whose behaviour and feedback required interpretation by the students. The strong link between spatial graphical and geometrical aspects helped students go beyond an immediate perceptual approach, conceive angle as turn and measure and connect it with physical angle situations (e.g., the limited rotation of a door) that usually cause special difficulties to young students. Thus turn was viewed not only as a relationship between the headings-faces of the turtle and its traces but also as a relationship between the planes defined by the 2d parallelogram drawn by the turtle.

Student's visualisation and construction strategies are inevitably related to the representational registers used and tool's affordances. This small scale research brought in the foreground issues related to software's semantics. The preference of students in working initially in the 2d vertical level should be related not only to the complexity of 3d space but also to students' viewing axis of the display and the endogenous difficulties of represented 3d objects in the 2d computer screen so as to look 3d. Although certain conventions are used in MaLT, so as to help students appreciate depth (the z co-ordinate), it seemed that they are not enough and that in some cases they can be even confusing. This finding stresses the need for the provision of further visual cues and conceptual hints so as to help students appreciate 3dness.

In this small scale research we formed the impression that MaLT can be a new resource for activity and construction of meanings where the mathematical formalism of 3d geometry and graphical representation of objects and relations can be dynamically jointed in interesting ways, strongly related to the representational infrastructure of MaLT. However further researches are needed were more attention should be given to tasks' design and classroom orchestration so as to investigate in depth and shed light upon the various interconnected mathematical concepts that are involved in 3d space visualisation and conceptualisation.

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Vėžliuko navigacija ir geometrinių figūrų valdymas trimatėje erdvėje, remiantis kintamais procesais

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Šiame straipsnyje aptariamos problemos, susijusios su Vėžliukų navigacija trimatėje erdvėje ir geometrinių figūrų valdymu, remiantis naujai sukurta aplinka, būtent, MaLT. Reikšmingų formalizavimų bendras naudojimas, kuris yra stipriai susijęs su MaLT atstovaujančia infrastruktūra ir grafiškai pateiktų erdvinių figūrų dinaminė manipuliacija, siūlo naujas galimybes objektams valdyti. Straipsnyje aptariami įvairūs šios temos sunkumai, aptariamos geometrinio veiklumo ir reikšmių konstravimo sąsajos.