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Animation Tools of CAS for Dynamic Exploration of Mathematics

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Abstract. The scope of the paper is animation facilities of computer algebra systems (CAS). Animation offers opportunities for visualization of complex mathematical concepts, provides convincing demonstration of ideas and influence of quantities or parameters, helps to generate hypothesis, encourages exploration. Animation can be used to demonstrate many mathematical concepts that are difficult to explain verbally or to show with static pictures. Using animation allows students to explore, experiment and visualize mathematics as a dynamic process. But CAS creates only opportunities. The problem remains for users to realize this potential. So features of CAS such as ease of use, convenience of procedures are important for teaching and learning. The paper deals with animation features of the three most popular CAS – Maple, Matlab, Mathcad and their usefulness in education. The results of practical use of the three CAS in teaching animation procedures are discussed and students' opinion about animation tools of CAS is presented.

Key words: computer algebra systems (CAS), visualization, animation.

1. Introduction

Visualization is very important in general cognitive skill acquisition and problem solving processes. As one of the more advanced visualization forms – animation offers opportunities for visualization of complex mathematical concepts, provides convincing demonstration of ideas and influence of quantities or parameters, helps to generate hypothesis, encourages exploration. Images, especially moving images, activate mental processes such as perception of spatial relationships, changes, complex processes or observation of patterns and therefore, help understanding. Using animation allows students to explore, experiment and visualize mathematics as a dynamic process. Animation can make mathematics more interesting and stimulating, more dynamic and meaningful. Mathematicians can now use computers to generate pictures that would be tedious or impossible to generate by hand. Since Ivan Sutherland's first work in 1963 on computer graphics (Biography of Ivan Sutherland, `www.sun.com/960710/feature3`), which later followed by 3-D computer modeling and visual simulations, and since the first symbolic algebra system to do animation – Mathscribe (`www.mathscribe.com`), many computer algebra systems (CAS) were created that use advanced computer graphics and animation for mathematical visualization. Mathematica (`www.wolfram.com`), Maple (`www.maplesoft.com`), Mathcad (`www.mathsoft.com`) and Matlab (`www.mathworks.com`) seems to be the most powerful "animators". Discussions and examples of animation created with CAS Mathematica are presented in (Gloor *et al.*, 2005; Kidron, 2003; Talman, 2004). A book (Putz, 2003) provides description of Maple animation tools, many examples of useful for mathematical education created with Maple vivid images are presented in (Heath, 2004). Generated with Mathcad lecture supplements for teaching Calculus are presented in (Bogacki, 2001; Peil, 2002). Matlab animation for numerical methods one can find in (Mathews, 2005). However, creating of animated images is nontrivial. Developing high quality educational multimedia content requires more than subject matter expertise – first of all, it demands a significant investment of teacher's time. Students also experience difficulties with new approaches, because use of CAS creates only opportunities. The problem remains for users to realize this potential. So features of CAS such as ease of use, convenience of procedures are important for teaching and learning. If one considers animation facilities in the CAS, they seems similar – all systems support visualization of drawing process itself, generation of moving images and surfaces, creation of stand-alone video files. But the animation tools are different in all systems, animated pictures as a result are given in different ways. The application ease is important as it makes learning enjoyable rather than boring and difficult. So the purposes of the article is to compare animation features in the three popular CAS – Maple, Mathcad and Matlab – from the point of view of application ease, convenience for student and possibilities to create animated images during short laboratory works. The main goals of the article are to investigate and to compare

- mechanisms of animation in the three CAS;
- ease of application of animation facilities;
- students opinion on application convenience of animation tools.

2. Mechanisms of Animation in CAS

Similar as in a creation of animated films, the common way to get animated graphs in mathematics is generation of serious of graphs and sequencing them in time creating effect of motion. Though the tools of animation in the three CAS are different, one can see that the main animation creating aspects are common. The basic animation facilities of the three CAS are shortly described in the Table 1. Each system has one or more its own ways to create moving images of curves and surfaces, to describe a process of function drawing and to generate stand-alone files, which can be used without special mathematical software (e.g., in Power Point presentations or illustrations of mathematical information on the web). More information on animation facilities in each computer algebra system one can find in books and web pages, presented in References. Let us look at basic animation facilities in the three systems.

As one can see in Table 1, the system **Maple** provides the easiest ways to create animated images. The *animate* function provides support for two-dimensional plots of one or more functions specified as expressions, procedures or parametric functions. For

Table 1

Basic animation facilities in the three CAS

Basic Animation Facilities	CAS	**Maple**	**Mathcad**	**Matlab**
	Moving images in 2D and 3D	Animated images are created using functions *animate*() and *animate3d*(), e.g., *animate*$(\sin(x*t), x=-4..4, t=0..4)$; *animate3d*$(\cos(t*x)*\sin(t*y), x=-$*Pi*..*Pi*, $y=-$*Pi*..*Pi*, $t=1..2)$. It is also possible to construct an animation sequence from existing plots via using the *display* function with the *insequence* option	1 *step*. Description of variables and of a function, including special parameter FRAME. 2 *step*. Plotting a static graph with FRAME=0. 3 *step*. Choosing animation parameters in special window, marking region of description and playing animation	1. *Movie-making frame by frame*. A number of figures is created and each one is stored as a frame, using ***getframe*** command. The movie then is played back with the ***movie*** command: *for* $j=1:n$ *plot command* $M(j)=$*getframe*; *end* *movie*(M) 2. *Erase mode.* Continually erasing and then redrawing the objects on the screen, making incremental changes with each redraw.
	Process of drawing	One or more curves are easily plotted in 2D using the function ***animatecurve***(), e.g., *animatecurve*$(\{x-x\hat{}3, \sin(x)\}, x=0..$*Pi*$/2)$	There is no special functions, a user has to program the process of drawing	One curve is easily plotted in 2D or 3D using ***comet***() or ***comet3***() functions, e.g., $x=-$*pi* $: 0.01 :$ *pi*; *comet* $(\sin(x)./x)$ $x=0:$ *pi*$/1000 : 6*$*pi*; *comet3* $(\cos(x), \sin(x)+x/10, x)$
	Creating of stand-alone files	***GIF or JPEG files***. For creating of stand-alone gif or jpg one have to choose meniu ***Export-Graphics Interchange Format*** (GIF) or ***Export- JPEG File Interchange Format*** (the graph have to be marked). Quick Time Player is needed for playing	***AVI files***. For creating of AVI files one have only to choose ***Save as*** in Animate dialog window (Fig. 2) and to select path for file saving. Microsoft Windows Media Player is needed for playing	***MPEG files.*** MPGWRITE program is needed for creating of graphic files of mpeg format. After you install this program, a command ***Mpgwrite(M, jet, 'filename.mpg')*** creates an animated stand-alone file, where ***M*** is movie matrix, ***jet*** is the default colormap. MPEG Movie Player is needed for playing

example, a dependence of the function $y = ax^2$ on the parameter a would be described after the command *animate*$(a * x\hat{}2,\ x = -4..4, a = -2..2)$. A graph of the function plotted then has to be marked and animation button pushed for playing animation.

In a three dimensional case a typical call to the *animate3d* function is *animate3d*$(f(x, y, t),\ x = a..b, y = c..d, t = p..q)$, where f is a real function in x, y, and t, and $a..b$ and $c..d$ specifies the range on which f is plotted while $p..q$ specifies how the frame coordinate varies from one frame to the next. So if an analytic expression of a function is available, one have just to type *animate* or *animate3d* function in a right way, describing changeable function and its parameters.

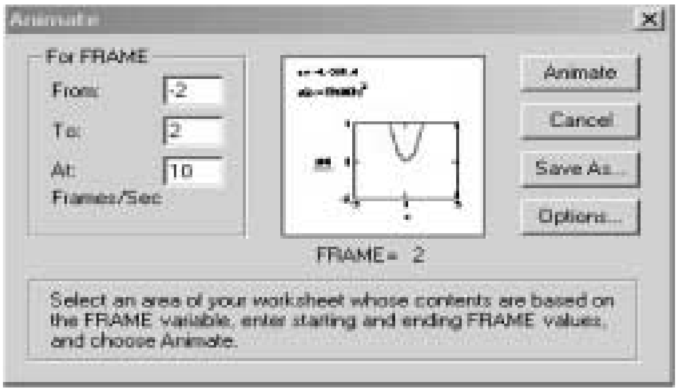

Fig. 1. Animation dialog box in Mathcad.

Constructing animations in **Mathcad** is a little more complicated and especially depends on student's experience to use the CAS itself. The same example – a moving picture of the function $y = ax^2$ would be created after

1) typing
   $x = -4, -3.99..4$,
   $y(x) =$ *FRAME* $* x^2$;
2) drawing the function graph;
3) opening *Animate* window (Fig. 1), choosing parameters, marking a region of description (where x and $y(x)$ are described together with a graph of the function) and clicking *Animate*.

So Mathcad creates a video by shooting a sequence of frames. One frame is generated, in order, for each value of system's parameter FRAME in the specified range. A user has to describe changeable function in a right way, including FRAME. The necessary next steps, as choosing of animation speed, selecting of description area and clicking *Animate* leads to the created animated picture of the function.

There are two basic methods for creating animation in **Matlab**: *Movie-making frame by frame* and *Erase mode*. According to the first, number of figures are created and each one is stored as frame, using *getframe* command. The movie is then played back with the *movie* command. The function *getframe* is usually used in a *for* loop to assemble an array of movie frames for playback using *movie* function, where m is a number of times to repeat animation:

for $j = 1 : n$
plot command
$M(j) =$*getframe*;
end
movie(M, m)

Animation of the same function $y = ax^2$ would be created and played five times by typing, for example,

for $a = -2 : 2$

$i = a + 3;$
plot$(a * x.\hat{}2)$
$M(i) =$*getframe*;
end
movie$(M, 5)$

The second way – *Erase mode* plots a graph of function with initial values, then erases, then plots the changed function graph, erases, and so on. Both methods require programming skills in Matlab.

Moving pictures of parametric curves, images in 3D, graphs in polar or spherical coordinate systems can be displayed as well. It is possible to describe changes of several functions together in one window, or in separate windows. One can see that more complicated animations need programming skills in each system.

All three systems also have opportunity to create stand-alone files, independent from CAS software, though every system needs different video players for playing animations. Fortunately, all of them are free and can be downloaded from the Internet.

Though the way of creating moving images are different, the results usually are similar. So our purpose was to use animation tools of all three systems in the classroom and to see complexity of application in practice.

3. Examples of Animation Use

Despite the problems of teaching mathematics with computer are not trivial and widely discussed in conferences and papers (Solomon and Schrum, 2002; Handal and Herrington, 2003; Lipeikienė, 2004; Lipeikienė, 2005), everybody agrees that visualization is always useful.

Animation first of all is visualization of some variations. Fig. 2 describes the use of animation in introducing a Gaussian density: first three fragments of animation demonstrate changing of the Gaussian density with a change of a variance, the last three – changing of a mean.

Animation also can help to introduce mathematical concepts, for example, geometrical and trigonometrical notions, such as ellipse (Fig. 3) or sine function (Fig. 4).

Other typical example of animation is varying Riemann sum in numerical integration (Fig. 5). Though the static images in Figs. 2–5 do not describe the real power of animation, but they are as a hints to mathematicians, where animation should be used: to describe some variations, ranges, approximations, transformations and other changes. Such topics of mathematics as

- definition of functions and changes of function parameters,
- rotating surfaces and curves,
- families of parametric curves,
- definition of derivatives,
- approximation of series by Taylor polynomials,
- illustration of definite integral and integral applications,

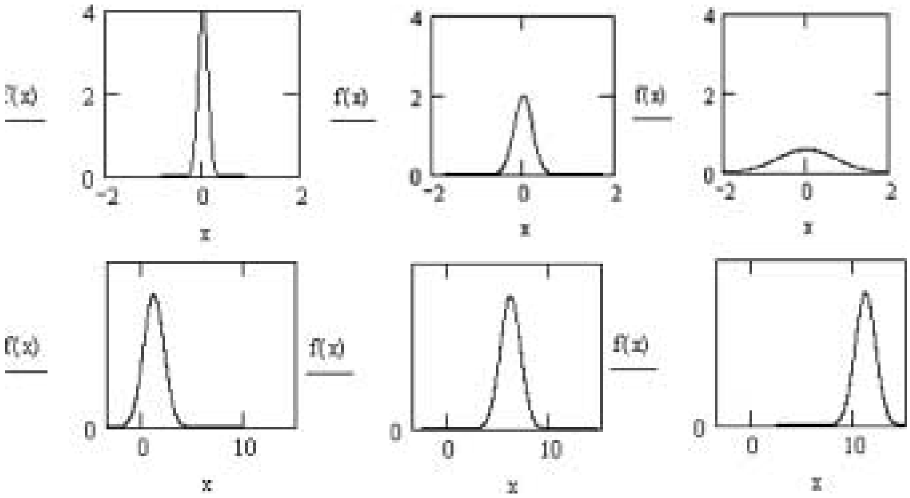

Fig. 2. First three fragments of animation demonstrate changing of the Gaussian density when a variance is changing, the last three – when a mean is changing.

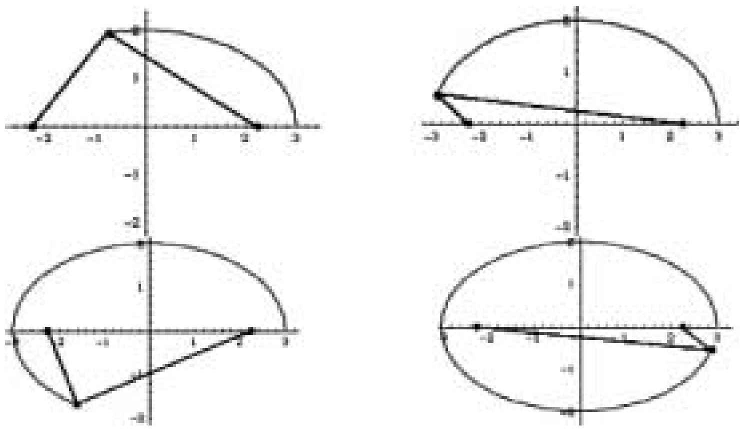

Fig. 3. Illustration of ellipse definition.

- numerical integration,
- definition of geometric concepts and dependencies etc.

are the typical topics where animation procedures should find their fascinating use.

The more advanced mathematical topics need more practices of animation creator, but CAS possibilities reduce efforts that are needed for creating moving pictures illustrating important features.

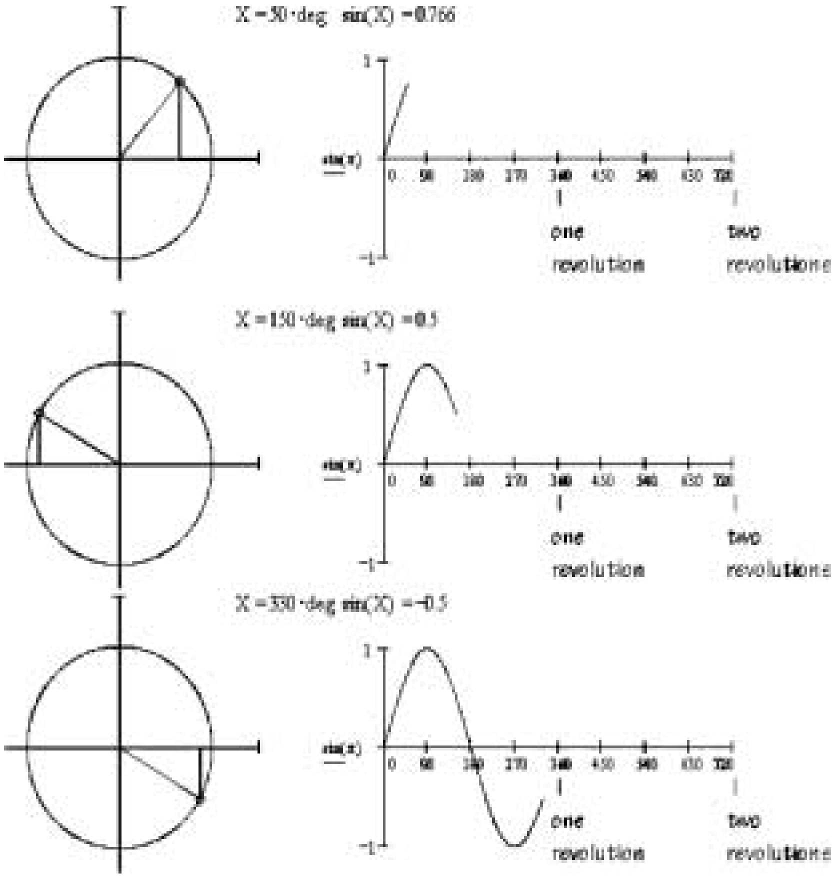

Fig. 4. Definition of sine function.

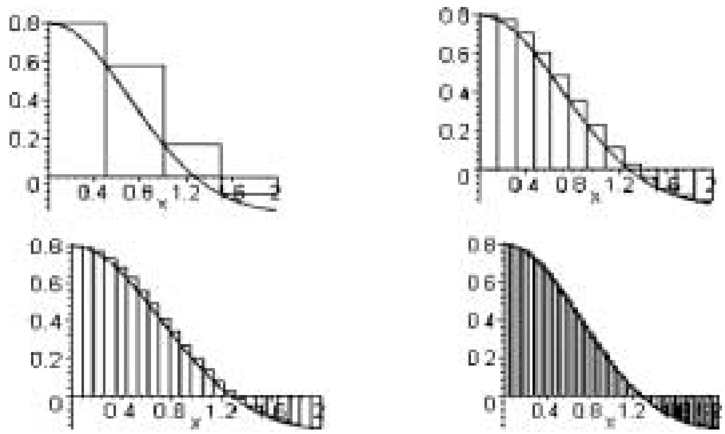

Fig. 5. Variation of Riemann sum, depending on the number of intervals.

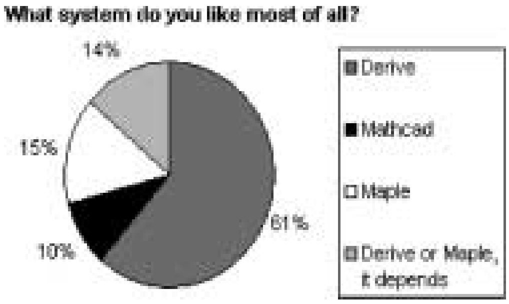

Fig. 6. General students' opinion about four CAS.

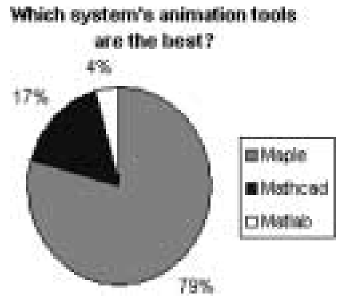

Fig. 7. Students' opinion about animation tools of the three CAS.

4. Investigation of Animation Tools with Students

72 second year students of mathematics and informatics speciality at the Vilnius Pedagogical University studied Computer Algebra Systems (CAS) course for one semester. They were taught problem solving with CAS Derive, Maple, Mathcad and Matlab. Special attention was paid to animation features of CAS in order to find students' opinion about application ease of animation facilities in the different CAS, to get to know, which system's animation tools are most acceptable for students. During all semester students created animations with the three CAS – Maple, Matlab and Mathcad during laboratory works and at home. They used animation in various mathematical topics. Watching the process of their studies it became clear that students were interested in possibilities of CAS to create moving images. They understood the power of animation in learning mathematics and most of them intended to use animation in teaching mathematics in the future. Also one could see Maple animation leadership: students tried all possibilities, but compared them with Maple and discussed Maple advantages. After the semester students filled in questionnaire about animation tools in CAS. Figs. 6–8 illustrate their opinion. Students worked with examples from mathematics fundamentals, and only some of them programmed more complicated animations.

So students prefer Derive as the best tool for learning basic mathematics, calculus, linear algebra. It was seen during lectures. When they worked with animations, it became clear that they preferred Maple animation tools, though some of them liked Mathcad.

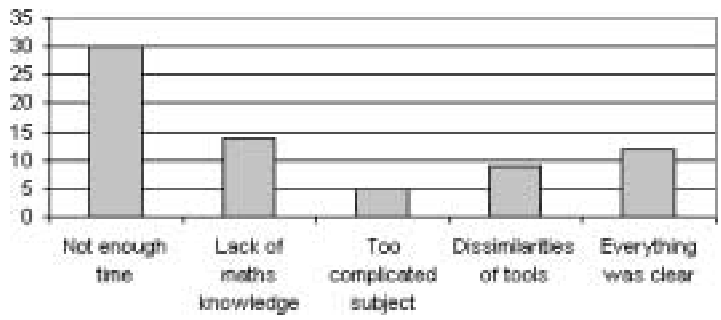

Fig. 8. Students' opinion about disadvantages in the learning of animation facilities.

Most of them (93%) would use animation for teaching mathematics at school, and to the question "What animation tools are the best for teaching?", 59% answered that Maple, 25% – Mathcad, only 1% – Matlab, and 15% – any available CAS with animation tools. Talking about drawbacks, which they met learning animation, students enumerated various disadvantages (Fig. 8). Obviously, most of the shortcomings are subjective, depend on students' mathematical capabilities.

5. Conclusions

The discussed CAS have useful animation facilities that enable demonstration of miscellaneous mathematical variations. Animation possibilities of CAS reduce efforts that are needed for creating moving pictures, enhance teaching and learning of mathematics. Especially simple procedures provide Maple, but more advanced topics require expertise and programming skills in all three CAS. Investigation of students' opinion about animation facilities in the three CAS (Maple, Mathcad and Matlab) showed that for creating animations most of students preferred to use Maple, but some of them liked Mathcad or Matlab. The best students do not see disadvantages in using any of available CAS with animation facilities. This was ascertained during laboratory works and evaluating questionnaire results.

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KAS animacijos įrankiai dinaminiam matematikos studijavimui

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Straipsnis nagrinėja kompiuterinės algebros sistemų (KAS) animacijos galimybes. Animacija siūlo sudėtingų matematikos sąvokų vizualizavimo galimybes, teikia matematinių idėjų ir parametrų įtakos įtikinamą demonstravimą, padeda generuoti hipotezes, skatina matematikos studijavimą. Animacijos gali būti naudojamos pademonstruoti daugelį matematikos sąvokų, kurias sunku paaiškinti žodžiu ar statiniais paveikslėliais. Naudodami animaciją, studentai gali mokytis, eksperimentuoti ir vizualizuoti matematiką kaip dinaminį procesą. Bet KAS teikia tik galimybes. Vartotojams reikia realizuoti šį potencialą. Tokios KAS savybės, kaip vartojimo paprastumas, procedūrų patogumas yra svarbios mokymui ir mokymuisi. Straipsnyje nagrinėjamos trijų populiariausių bendrųjų KAS – Maple, Mathcad ir Matlab animacijos savybės ir jų naudingumas mokymui. Aptariami praktinio šių sistemų naudojimo, mokant studentus kurti animacijas iš įvairių matematikos sričių, rezultatai ir studentų nuomonė apie šių trijų KAS animacijos įrankius.